THE [illegible] ELECTRONIC [illegible]

IN THE DETERMINATION [illegible]

OF [illegible]

TABLE OF CONTENTS

[illegible]

[illegible] purp[illegible]
[illegible]tion of [illegible]
[illegible] a ther[illegible]
[illegible] consid[illegible]
[illegible] lectin[illegible]
[illegible]
[illegible] type [illegible]
[illegible]
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[illegible]
[illegible] consider[illegible]
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[illegible] of standard [illegible]
[illegible]nctions are [illegible]
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[illegible]
[illegible] close appro[illegible]
[illegible]phical methods [illegible]
[illegible] considering [illegible]
[illegible] and rotary [illegible]
[illegible] the assumptions [illegible]
[illegible]tion.

This investigation [illegible] conducted [illegible] tories of the Aeronautical Engineering Department [illegible]

University of Michigan, [illegible] requirements
for Master Degrees, by [illegible] and G. S. P[illegible]

THE USE OF [illegible]

IN THE [illegible]

OF LATERAL VIB[illegible]

INT[illegible]

Solutions of [illegible] mechanical [illegible] analogs for some [illegible] the electrical [illegible] terms due to variation [illegible], with the [illegible] amplifier [illegible] techniques [illegible] computer [illegible] computed by [illegible] pitch [illegible]. One [illegible] analog computer [illegible] for the [illegible] of Naval [illegible]

The techniques [illegible] stabilized [illegible] after the [illegible] G. E. [illegible] and R. [illegible] electronic computers [illegible]

[illegible] this method, [illegible]

[illegible] following investig[illegible] [illegible]

[illegible] electronic [illegible]

Several simpler p[illegible]

[illegible] the first solved [illegible]

[illegible] with the use of [illegible]

[illegible] the simplicity [illegible]

[illegible] complicity [illegible]

[illegible] these [illegible]

[illegible] the third [illegible]

[illegible] dimensio[illegible]

[illegible] the first four [illegible]

[illegible] the effects of [illegible]

[illegible]

[illegible] gratitude to [illegible] Professor [illegible]

[illegible] Acknowledgments [illegible]

[illegible] University of Michigan [illegible]

[illegible]

As stated in [illegible]

[illegible] necessary [illegible]

[illegible] were provide[illegible]

[illegible] equipment [illegible]

[illegible] are to be [illegible]

[illegible]

[illegible] of the [illegible]

[illegible]

The unit [illegible]

[illegible]

[illegible] stability [illegible]

[illegible] of 110 [illegible]

[illegible] the amplifier [illegible]

[illegible] changes, and [illegible]

[illegible] any of these [illegible]

to make external chang[illegible]

[illegible] and feedback circ[illegible]

[illegible] of the [illegible]

[illegible] photograph of the [illegible]

[illegible] Fig. [illegible]

[illegible] of [illegible]

[illegible] Due [illegible]

the amplifier it change[illegible]

[illegible] in every operation. [illegible]

input and feedback impedances consist of equal resistors; for multiplication the desired ratios of feedback to input resistors are plugged into the amplifier circuit.

The amplifiers are connected to a power supply distribution panel by means of [illegible]-wire shielded cables. The two knobs shown on the chassis of the amplifier are used to balance the amplifier for zero direct current output prior to use in the computer circuit. The knobs are connected to two variable resistors associated with the [illegible] tube.

To balance a multiplying or sign changing amplifier, the input is shorted to ground and equal resistors (one megohm) are plugged into the input and feedback circuits. By use of a multi-range direct current vacuum tube voltmeter connected between ground and the output, zero output is obtained by adjusting the two knobs.

The procedure for balancing an integrating amplifier is similar. [illegible] and [illegible] adjustments are [illegible] the knobs for [illegible] output [illegible].

The procedure for balancing [illegible] amplifiers in [illegible] computer [illegible] differ [illegible] depending on the problem [illegible]

[illegible] through [illegible]

acceptable over-all [illegible] the [illegible]

[illegible] amplifiers [illegible]

[illegible] with the [illegible]

The [illegible]

are designed for [illegible]

[illegible]

[illegible] feedback circuits [illegible]

[illegible]

[illegible] leakage resist[illegible]

The high voltage [illegible]

[illegible] consist of [illegible]

[illegible] regulated [illegible]

[illegible] These voltages [illegible]

[illegible] the amplifier [illegible]

[illegible] distribution box [illegible]

[illegible] distribution box [illegible]

[illegible] of amplifier [illegible]

[illegible]

[illegible]

[illegible] on [illegible]

[illegible] could [illegible]

[illegible] one [illegible]

[illegible] relay control [illegible]

[illegible] the computer [illegible]

[illegible] stepping relay [illegible]

[illegible] minimise the [illegible]

[illegible] on the [illegible]

The stepping [illegible]

[illegible] Each relay [illegible]

[illegible] each. [illegible]

[illegible] while the [illegible]

[illegible] is used [illegible]

[illegible] while one of [illegible] used to [illegible]

[illegible] series connected [illegible] on the [illegible]

With this equipment it is [illegible] possible [illegible]

[illegible] a variable [illegible]

[illegible] stations along the [illegible]

[illegible] to [illegible]

[illegible] per second [illegible]

[illegible] in terms of seconds. [illegible]

[illegible] the [illegible]

[illegible]

[illegible] the [illegible]

[illegible] can [illegible]

[illegible] is built [illegible] [illegible] ing the length.

[illegible] stepping relay [illegible] [illegible] control three such [illegible] [illegible] majority of the initial conditions [illegible] [illegible] The following [illegible] [illegible] from Rel. [illegible]

[illegible] is the most [illegible] relay, [illegible] depending upon the [illegible] [illegible] on the [illegible]

[illegible], through normally closed contact [illegible] relay F [illegible] relay A [illegible] [illegible] no longer [illegible] [illegible] Relays [illegible] the [illegible] stepping relays B [illegible]

These three relays [illegible] part in imposing [illegible] [illegible] stepping rel[illegible] [illegible] the [illegible] [illegible] initial conditions [illegible]

[illegible] relay [illegible]

[illegible] relay E is [illegible] [illegible] are energized, [illegible] conditions. Relay E is [illegible] by [illegible] contacts on relay M. If relay M [illegible] it remains closed [illegible] contacts. [illegible] normally closed contacts [illegible] is closed, relay [illegible] are removed.) [illegible] relays on contacts [illegible] initial conditions [illegible] are not [illegible] which is done [illegible] every one [illegible] reaches contact [illegible]

The stepping relays [illegible] are in this [illegible] no longer [illegible] stepping [illegible]

[illegible]

[illegible] found to be [illegible] below [illegible]

"sticking" contacts of [illegible] the [illegible] the stepping relay [illegible] contact [illegible] remains closed. [illegible] conditions [illegible]

In case any one of [illegible]

[illegible]

[illegible] given in the following [illegible]

[illegible]

[illegible] preliminary to [illegible] investigation, three [illegible] of the [illegible] single beam [illegible] analog computer. Such [illegible] part of [illegible] program of [illegible] techniques to be used [illegible] setting up and operating [illegible] electrical circuits [illegible] network [illegible]

[illegible]

The first preliminary [illegible] tion of the static deflection [illegible] laterally supported [illegible] section which is small [illegible] different types of [illegible] , [illegible] and hinged.

[illegible] differential [illegible] is given [illegible]

$$EI\frac{d^4y}{dx^4} = q(x).$$

[illegible] solution:

$$y(x) = \frac{W(x)}{12\ EI}\left(\frac{L^2}{2}x^2 - Lx^3 + \frac{x^4}{2}\right)$$

$$y(max) = \frac{W(x)L^4}{384\ EI} \quad @ \ x = \frac{L}{2}$$

[illegible] (1). The [illegible] made [illegible] seconds, [illegible] such as [illegible] seconds.

Thus $x = \frac{L}{T}t$, and $\frac{d^n(\ )}{dx^n} = \frac{T^n}{L^n}\frac{d^n(\ )}{dt^n}$

[illegible] equation is then

$$EI\frac{d^4y}{dt^4} = \frac{L^4}{T^4}W(x)$$

The computer circuit [illegible] is given in Fig. [illegible] $y'(0) = 0$ [illegible] [illegible]

[illegible] and VI [illegible]

11372

[illegible] varied until a [illegible] [illegible] battery voltage [illegible] [illegible] to A_1 to simulate [illegible] [illegible], and was [illegible] in terms of [illegible] [illegible] in units. The [illegible] of A_4 and [illegible] [illegible] through amplifiers [illegible] [illegible] recorder [illegible] oscillograph of [illegible] and y.

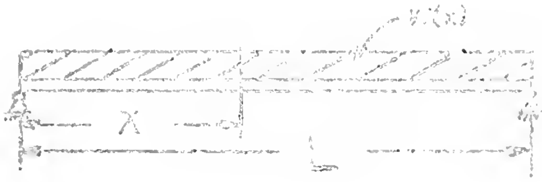
[illegible] relations [illegible] in the following [illegible] with a constant [illegible] [illegible] applied to A_1, [illegible] an arbitrary [illegible] [illegible] by closing [illegible] [illegible] The problem [illegible] initial conditions [illegible] the end conditions [illegible] with different settings of the potentiometer [illegible] the battery [illegible] [illegible] were [illegible] oscillograph. Fig. 11 [illegible] [illegible] oscillograph [illegible]

[illegible] $y(\ldots)$ [illegible]

$y(\max)$ theoretical [illegible]

$y(\max)$ from Fig. 11 [illegible]

[illegible] Beam Hinged at [illegible] ends



[illegible] Conditions: Beam [illegible] at end [illegible] are [illegible]

$y(0)$ [illegible]

Theoretical solution: 9

$y(\max)$ [illegible]

The computer equation [illegible] a change is necessary [illegible] be the change in [illegible]. The [illegible] for the solution [illegible]. The end conditions $y(\ )$ [illegible] by shifting [illegible]. These [illegible] from both ends of the [illegible] battery voltages $-Va$ and Vb [illegible]

[illegible] in [illegible] [illegible] [illegible] before for [illegible] [illegible] necessary in varying the [illegible] to [illegible] the proper end conditions.

Fig. 13 shows the solution of the [illegible] on the oscillograph. The length of the [illegible] on the oscillograph is measured [illegible] = 1.?.

[illegible]

ω (nor) theoretical = [illegible] = 1.30

ω (nor) from Fig. 13 = 1.15

The second preliminary problem was the determination of the first three natural modes of lateral vibration of a uniform free-free beam considering the effects of bending distortion only. The vibrating beam is considered loaded by inertia forces due to its own mass and acceleration.

The differential equation of motion of the elastic curve of such a beam is given by [illegible]

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} = -\mu \frac{\partial^2 y(x,t)}{\partial t^2} \qquad (1)$$

where $\mu = \frac{A\gamma}{g}$, the mass distribution along the beam

γ = the density of the material of the beam.

μ = ... mass ... of the beam

g = ...

$\mu \frac{\partial^2 y(x,t)}{\partial t^2}$ = the inertia ... acting ...

y = vertical deflection of the ...

x = distance along the beam ... from one end.

E = Young's modulus of ... material

I = Area moment of inertia of ... section of the beam with ... the centroidal axis.

It is assumed that $y(x,t) = X(x)e^{j\omega t}$, where $X(x)$ is a function ... of x, ... the beam, $e^{j\omega t}$ represents ... oscillation ... frequency ω.

Then
$$\frac{\partial^2}{\partial t^2} y(x,t) = -\omega^2 X(x) e^{j\omega t}$$

and equation (3) becomes

$$\frac{EI d^4 X}{dx^4} - \mu\omega^2 X = 0$$

or

$$\frac{EI}{\mu\omega^2}\frac{d^4 X}{dx^4} - X = 0 \qquad (4)$$

The computer equation is set up as ... by making a change of the independent variable ... equation (4). The independent variable ... is changed to t, time in seconds, and the length of the beam is expressed as T, total elapsed ... solution in seconds.

Then $x = \frac{L}{T} t$ and $\frac{d^n(\ )}{dx^n} = \frac{L^n}{T^n} \frac{d^n(\ )}{dt^n}$

The computer equation becomes

$$\frac{EI\ T^4}{\mu\omega^2 L^4} \frac{d^4X}{dt^4} - X = 0 \tag{5}$$

For simplicity we let

$$\alpha_n^2 = \frac{\mu\omega_n^2 L^4}{EI}$$

$$\omega_n = \alpha_n \sqrt{\frac{EI}{\mu L^4}}$$

where [illegible] is the [illegible]

[illegible]ency of vibration for the n^{th} mode. In [illegible]dition, for the computer equation (5) we [illegible]note

$$C = \frac{T^4}{\alpha_n^2}$$

[illegible] the computer equation reduces to

$$C \frac{d^4X}{dt^4} - X = 0 \tag{6}$$

For the computer solution, the C was given a value of unity (1 megohm) and the problem was solved by [illegible]ing a length, T, on the oscillograph [illegible] for which the simulated end conditions as determined by the beam supports were met.

A computer [illegible] solution of Equation (6) is given in Fig. [illegible]. The end conditions to be satisfied for a free-free beam are that the bending moment and shear force at each end are zero. These boundary conditions are expressed as

$$X''(0) = X'''(0) = X''(L) = X'''(L)$$

To satisfy these end conditions on the computer, the feedback capacitors of A_1 and A_2 are initially shorted. As there is a finite but unknown slope and deflection at each end of the beam, these are simulated on the computer by battery voltages $-Va$ and Vb respectively initially applied to the [illegible] through A_3 and A_4. As before, Vb was fixed at [illegible] volts and $-Va$ was varied for different trial solutions until the end conditions of zero shear force and bending moment were satisfied.

The outputs of A_1 and A_2 were connected through amplifiers to the two channels of the [illegible] recorder for recording oscillographs of X'' and X'''. Correct solutions showing the fulfillment of the end conditions required were obtained when [illegible] maxima of X'', depending upon the [illegible] of the mode, passed through the zero axis. The X''' curve was used in measuring the length, T, of the solution. This curve was used in preference to X'' because the X''' curve has a definite finite slope at the end of the solution.

Correct solutions of the [illegible] for the first three normal modes of vibration of the [illegible] were obtained in a manner similar to that previously described. Vb was made constant and -Va was initially set at an arbitrary value. All initial [illegible] conditions were imposed by closing the initial condition relay [illegible]. The problem was started by simultaneously releasing all the end conditions. Several trial settings of the potentiometer controlling the voltage -Va were necessary before a correct solution for each mode was obtained.

Solutions of the first mode were quite readily obtained, but for the second and third modes the setting of the potentiometer controlling the voltage -Va was found to be very critical. This fact and slight instability of the amplifiers used and [illegible] in the power supply voltage were enough to cause [illegible] of solutions. Many trials were necessary to obtain a few correct solutions. Fig. 11 shows the correct solution [illegible] obtained for the first three modes. The results obtained [illegible] agreed very closely with those given in the appendix of Den Hartog[illegible]:

	Mode	[illegible]	[illegible]
α_1	1	[illegible].4	[illegible]
α_2	2	6[illegible].7	[illegible]
α_3	3	121.0	[illegible]

[illegible] to synchronize [illegible] should be carefully [illegible]. [illegible] effectively reduced by increasing [illegible] for input voltage recorded on the oscillo[illegible].

7. The potentiometer controlling the [illegible] voltage [illegible] had three degrees of [illegible] found quite [illegible] of the [illegible]. [illegible] to the [illegible] above.

It was felt that the [illegible] of the preliminary [illegible] the laboratory [illegible] on the [illegible] balance [illegible] obtain [illegible] the [illegible].

Part III

For the purpose of this study, the [illegible] ship be replaced by an ideal [illegible] beam elastic and inertia properties. The [illegible] equation for the vertical motion of the [illegible] of this floating beam has been derived[illegible] including the effects of bending and shear deflection, rotary inertia, external loading, damping force, and buoyancy

[illegible]

[illegible] of the differential [illegible]

[illegible] the elastic curve [illegible]

[illegible] bending deflection [illegible]

[illegible] shear deflection [illegible]

The complete differential [illegible] terms are given by[11]

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 [illegible]}{\partial x^2} - I\mu\frac{\partial^2 [illegible]}{\partial t^2}\right] [illegible]$$

$$\left[\mu\frac{\partial [illegible]}{\partial t^2}\right] \quad [illegible]$$

[illegible] each term [illegible]

The first term

$$\left[EI\frac{\partial^2 [illegible]}{\partial x^2} - I\mu\frac{\partial^2 [illegible]}{\partial t^2}\right]$$

[illegible] be [illegible] of the [illegible] due to [illegible]

in bending and rotary inertia [illegible]

operator $\frac{\partial^2}{\partial x^2}$ reduces this to [illegible]

[illegible]

The term $\mu\frac{\partial^2 [illegible]}{\partial t^2}$ is [illegible]

[illegible] inertia.

Considering first the effect of [illegible]

only, the differential equation reduces [illegible]

$$\frac{\partial^2}{\partial x^2}\left[EI\,\frac{\partial^2 y(x,t)}{\partial x^2}\right]+\mu\,\frac{\partial^2 y(x,t)}{\partial t^2}=0 \qquad (3)$$

where μ = the mass distribution along the beam and includes the virtual mass which is an equivalent mass added to that of the ship to represent the inertia effect of the water associated with the ship's vibration.

For sinusoidal oscillations of frequency ω, it is assumed that $y(x,t) = X(x)\,e^{j\omega t}$

where $X(x)$ is a function only of x distance along the beam and is independent of time

$e^{j\omega t}$ represents sinusoidal oscillations of frequency ω.

Then $\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 e^{j\omega t}\,X(x)$

and assuming that E is a constant and that I and μ are functions only of x along the beam, equation (3) becomes

$$\frac{d^2}{dx^2}\left[I\,\frac{d^2X}{dx^2}\right]-\mu\omega^2 X = 0 \qquad (4)$$

We let $I = I_o i(x)$ and $\mu = \mu_o m(x)$ where I_o and μ_o are maximum values of moment of inertia and mass respectively.

To set up the computer equation, a change in the independent variable is again necessary. Where x was the independent variable, let t, time in seconds

[illegible] of the length [illegible] the

length of the ship [illegible]

[illegible] before, [illegible]

In general, $\frac{d^n( )}{dx^n}$ [illegible]

and [illegible] (C) [illegible]

[illegible]

$$\omega_n = \alpha_n \sqrt{\frac{EI_o}{\mu_o L^4}}$$

[illegible] natural frequency [illegible]

The computer equation [illegible]

$$\frac{d^2}{dx^2}\left[I(x) \frac{d^2 Y}{dx^2} \right] [illegible] = 0$$

In this computer [illegible]

[illegible] pro[illegible] [illegible] [illegible]

[illegible] to $\frac{d}{dt}\left[[illegible] \right]$

[illegible] ship rets [illegible]

[illegible] boundary condition [illegible] the problem [illegible]

[illegible] bending moment and shear [illegible]

The boundary [illegible]

$$[illegible] \frac{d}{dt}\left[f(s) [illegible] \right] [illegible] \frac{d^2 Y}{dt^2} [illegible] \left[[illegible] \right] [illegible]$$

The computer circuit for the solution of equation [illegible] is given in Fig. 11. The [illegible] bending moment and shear [illegible] by initially shorting the feedback [illegible] respectively. [illegible] each end of the beam are unknown, these [illegible] the computer by battery voltages [illegible] respectively [illegible] initially to the capacitors of [illegible] [illegible] at about [illegible] different trial solutions for the [illegible] the end conditions of zero shear and bending moment were satisfied.

In Table I is tabulated the [illegible] the [illegible] the ship for [illegible] frequency of [illegible] desired. This ship has the following [illegible] characteristics [illegible]

L = Load water line length [illegible] = [illegible]

B = Molded breadth = 50'

D = Molded depth 27'

d = Full load draft 15' [illegible]

Δ = Full load displacement [illegible] = [illegible] tons

[illegible] = Young's modulus of elasticity = [illegible] $tons/ft^2$

[illegible] = [illegible] moment of inertia [illegible] [illegible] ft^4

μ_0 = [illegible] = [illegible] tons sec^2 / ft^2

For the purpose of calculation, the ship was divided into 20 parts [illegible] length. For each section there is [illegible] the curve [illegible] and mass. Fig. 17 shows the distribution [illegible] moment of inertia [illegible] the curves. On table [illegible] and [illegible] which were [illegible] [illegible] [illegible]. The [illegible] [illegible]

[illegible]

At the stepping relay resistor panel, Fig. 6, as originally planned, the length of solution on the computer should have been T = 10 seconds. However, as the stepping relays used stopped the problem immediately upon reaching step 40, the length of the solution, T = 9.75 seconds. This condition could have been corrected by rewiring the stepping relays' control relay circuit to provide for a quarter second pause on step 40 before the initial condition relays again imposed the end condition and stopped the problem. The authors of this paper did not feel at liberty to change the equipment in this manner, and felt that the quarter second lost could be accounted for in the solution knowing that T was actually 9.75 seconds. Even though the quarter second in the length of the computer solution meant ten feet in the length of the ship, this "lost" section of the hull does not materially affect the vibratory characteristics of the vessel.

Fig. 9 shows schematically the arrangement of the computer and complete network of controls and power supply. Fig. 8 is a photograph of the complete network. The outputs of A_2 and A_3 are shown connected through amplifiers to the two channels of the Brush recorder for recording oscillographs of [illegible] and $-i(t)\,\frac{d^2X}{dt^2}$.

[illegible] of the beam.

[illegible] equation (9) [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] an approximation to the [illegible] following equation:

[illegible]

[illegible] it is [illegible]

[illegible] $\frac{[illegible]^2}{L^{[illegible]}}$, equation [illegible]

$$[illegible] \frac{[illegible]}{[illegible]}\left[i(t) \frac{[illegible]}{dt^2} \right] + E \frac{d^2}{dt^2}\left[[illegible] \right] = [illegible]$$

The natural frequen[illegible] [illegible] the [illegible] is [illegible]

$$\omega_n = [illegible] \sqrt{\frac{EI[illegible]}{\mu [illegible]}}$$

In order to obviate [illegible] circuit to [illegible] [illegible] variables, $i(t)$ [illegible], it [illegible] could be [illegible] a represen[illegible] product term $\left[i(t)\ \beta(t) \right]$ [illegible] beam [illegible] the length of the [illegible], [illegible] with the regrouping of constants [illegible] equation (15) to a single con[illegible]

$$D = E \left[i(t)\ \beta(t) \right]$$

and the c[illegible]ter equation bec[illegible]

$$[illegible] \left[i(t) \frac{[illegible]}{dt^2} \right] + D \frac{d^2x}{dt^2} [illegible] \beta(t) [illegible]$$

([illegible])

The [illegible] conditions [illegible] bending moment and shear are [illegible] at both [illegible]. In the particular problems discussed, [illegible] the second and third derivati[illegible] of the deflection respectively. When in addition [illegible] bending deflection

[illegible] inertia [illegible]

[illegible] and shear [illegible]

$$M = EI\left(\frac{\partial^2 y}{\partial x^2} + \frac{\mu \omega}{EI} [illegible]\right)$$

$$[illegible] = \frac{EI}{\frac{\mu I \omega^2}{KAG}} \left[\frac{\partial^{[illegible]}}{\partial x^{[illegible]}} [illegible] \frac{\mu [illegible]}{[illegible]} [illegible] \frac{\mu \omega}{[illegible]} \frac{\partial [illegible]}{\partial [illegible]}\right].$$

[illegible], in setting up the [illegible] equation [illegible] [illegible] was made that $\frac{\partial^2}{\partial t^2} \frac{I\mu}{KAG} \frac{\partial}{\partial [illegible]}$ [illegible] neglected for [illegible] [illegible] shear and rotary [illegible] [illegible], the bending moment is proportional to [illegible] [illegible] the shear is proportional to

$\frac{[illegible]}{[illegible]}\left[[illegible]\right]$ in the computer equation ([illegible])

[illegible] boundary conditions for the solution of the computer [illegible] are then expressed as

$$[illegible] \frac{d^2 [illegible]}{[illegible]} = \frac{[illegible]}{dt}\left[I(o) \frac{d^2 [illegible]}{dt^2}\right] = [illegible] \frac{d^2 [illegible]}{dt^2} = \frac{d}{dt}\left[[illegible] \frac{d^2 [illegible]}{dt^2}\right] = 0$$

The computer circuit for the solution of equation ([illegible]) is given in Fig. [illegible], where the dotted [illegible] with resistance [illegible] from the output of [illegible] to the input of $A_{[illegible]}$ is included to accomplish the [illegible] $\frac{d^2 [illegible]}{dt^2}$ term in the computer equation. The procedure for the [illegible]-

[illegible] ration (1 [illegible]) [illegible] solution of equation (1) [illegible] [illegible] [illegible] [illegible]ly more [illegible] [illegible] [illegible] It is now [illegible] [illegible] [illegible]. [illegible] remains constant for all [illegible] A_4 could have been [illegible] [illegible] instead of C/D [illegible] variable introduced as a [illegible] [illegible] from trial to [illegible] [illegible].

The oscillographs of [illegible] [illegible] first four modes are given in [illegible] [illegible].

It was found that [illegible] [illegible] to A_2 from A_4 [illegible] [illegible] the computer circuit in the [illegible] [illegible]. The alternative of [illegible] a [illegible] as mentioned above as an input [illegible] [illegible] successful in stabilising the circuit and [illegible] used.

Theoretical solutions of the frequency of [illegible] modes of vibration of the [illegible] have been [illegible] by a calculator designed by [illegible] for [illegible] and shear deflection and rotary inertia effects, [illegible] results are listed below.

Frequency of Vibration - [illegible] [illegible]

Bending and Shear Deflection and Rotary Inertia

Mode		Computer	[illegible]
	C	rad/sec	rad/sec
1	7.00	11.99	10.2[illegible]
2	1.6[illegible]	23.42	19.[illegible]
3	0.7[illegible]5	35.00	30.0[illegible]
4	0.4[illegible]	46.[illegible]0	[illegible]

The frequencies obtained by [illegible] of the analog computer when the effects of bending deflection only are considered are approximately five to eight percent higher than the [illegible] solutions. The analog computer results are calculated directly from the oscillographs and have [illegible] [illegible] possible [illegible] [illegible] or power supply frequency variation. The latter is an important factor, and [illegible] should be made for better accuracy of results. Line frequency variation has an effect both on the measurement of [illegible] on the recorder tape and on the balance of the operational amplifiers, necessary for repetition of solution.

Another source of error in the computer affecting the accuracy of the results lies in the [illegible] resistor panels used. Every effort was made to [illegible] accurate resistance in each step for the simulation of mass and moment and inertia. However, the

many plug-in connections of resistors in stacks on the resistor panels introduced inaccuracies in the actual resistance obtained for each step. Then too, it is known that the bridging contacts of the stepping relays did not always perfectly bridge from one step to the next.

It is felt that the results were well within the accuracy of the computer network itself, and that corrections made to the oscillograph records as mentioned above would improve the precision of the values of frequencies of vibration obtained.

The results obtained when, in addition to bending deflection effect, an approximation to the effects of shear deflection and rotary inertia was considered are progressively higher, (from about ten percent for the first mode to about eighteen percent for the fourth mode) than those obtained by the [illegible] computer.

It is apparent that these deviations which increase with the higher modes result from something more than the inaccuracies in the computer network. The assumptions made to approximate the effects of shear deflection and rotary inertia in setting up the computer equation must, in a large part, account for the increasing error. These assumptions were necessary to avoid complicating the present computer network to a degree out of proportion to the actual effect of shear and rotary inertia on the frequencies

vibration of the ship.

With a computer constructed using [illegible] multiplying components, the product term $[I(t)\beta(t)]$ could be introduced as a variable. A network could be set up to solve a computer equation which includes the term

$$\frac{\partial^2}{\partial t^2}\frac{I_x}{KAG}\mu\frac{\partial^2 y(x,t)}{\partial t^2}.$$

The precision of the values of frequencies obtained from such an analog computer network should be definitely better than in the present instance.

CONCLUSIONS

Solutions to many engineering problems of practical interest involving higher order differential equations with variable coefficients may be obtained by means of a relatively simple and inexpensive electronic analog computer. Solutions so obtained are well within the accuracy necessary for most engineering purposes.

The accuracy of solutions obtained are limited by the precision of the computer components used and regulation of the associated power supplies. The assumptions made in reducing an exact differential equation to a computer equation are in a large part necessitated by the precision

[illegible] mode. [illegible] persons [illegible] would [illegible] introduced into the computer [illegible].

[illegible] desired accuracy [illegible] quantity of the computer increases, [illegible] and reliability of the components [illegible].

Instead of introducing another variable of the longitudinal motion into the [illegible] equations, additional amplifiers [illegible] constant average effect of [illegible] is obtained. Errors resulting from [illegible] of average effect must be weighed against [illegible] resulting from lack of precision of [illegible] elements. In the case of the [illegible] the constant average effect of [illegible] and rotary inertia embodied in the term [illegible] would appear not to be representative of the true effect. However, if the term $\frac{\partial^2}{\partial t^2}$ [illegible] had been retained in setting up the computer equations, there would undoubtedly have been closer agreement in results obtained. In the latter case the true effect was assumed to be [illegible], an obvious over-simplification in light of the [illegible] of the computer.

Economy of time and personnel is [illegible] benefit of such an analog computer [illegible] this paper. With the [illegible] familiarity with the operating [illegible] the solution of higher order differential equations with variable coefficients would be a matter of [illegible] for a single operator.

The analog computer is especially adaptable to the solution of design problems where the study of the effects of varying design parameters may be evaluated with little effort by simple external changes to a simple computer network.

[illegible]

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C A L C U L A T I O N S

CALCULATION OF FREQUENCY OF THE [illegible]

Bending Only

Constants used:

[illegible] = [illegible].75 [illegible] $T^2 = 95.0625 \ sec^2$

$$\sqrt{\frac{EI_o}{\mu_o L^4}} = \sqrt{\frac{1.[illegible] \times 10^5 \times 2[illegible]}{2.006 \times 256 \times 10^{[illegible]}}} = 0.314$$

1st Mode

$C = 6.0$

$$\alpha_1 = \frac{T^2}{\sqrt{C}} = \frac{95.0625}{\sqrt{6.0}} = 38.[illegible]$$

$$\omega_1 = \alpha_1 \sqrt{\frac{EI_o}{\mu_o L^4}} = 38.85 \times 0.314 \text{ rad./sec.}$$

$\omega_1 = 12.4$ rad/sec.

2nd Mode

$C = 0.98$

$$\alpha_2 = \frac{T^2}{\sqrt{C}} = \frac{95.0625}{\sqrt{0.98}} = 95.1$$

$$\omega_2 = \alpha_2 \sqrt{\frac{EI_o}{\mu_o L^4}} = 95.1 \times 0.314 \text{ rad./sec.}$$

$\omega_2 = 29.[illegible]$ rad./sec.

3rd Mode

$C = 0.25$

$$\alpha_3 = \frac{T^2}{\sqrt{C}} = \frac{95.0625}{\sqrt{0.25}} = 190.125$$

$$\omega_3 = \alpha_3 \sqrt{\frac{EI_o}{\mu_o L^4}} = 190.125 \times 0.314 \text{ rad./sec.}$$

$\omega_3 = 60.6$ rad./sec.

[illegible]

[illegible]

[illegible]

[illegible] $T^2 = $ [illegible] sec.^2

$\dfrac{EI_0}{\mu_0 l^4} = $ [illegible]

From Table XIII [illegible] $= $ [illegible] $\left[\phi(x)\ \beta(x) \right]$

$T = \dfrac{\text{[illegible]}}{\text{[illegible]}} = \dfrac{\text{[illegible]}}{\text{[illegible]},000}$

$= $ [illegible] sec.^2

$= $ [illegible] $\left[\text{[illegible]} \right]$ [illegible]

$= $ [illegible] sec.^2

[illegible]

[illegible]

[illegible] $= \dfrac{\text{[illegible]}}{7} = $ [illegible]

$\omega_1^2 = \dfrac{\text{[illegible]}\ EI_0}{\mu_0 l^4} = 12.1 \times $ [illegible]

$\omega_1 = 11.29$ rad./sec.

[illegible]

[illegible]

[illegible]

$\omega_2^2 = \dfrac{\text{[illegible]}\ EI_0}{\mu\ \text{[illegible]}} = $ [illegible]

$\omega_2 = $ [illegible] 02 rad./sec.

[illegible]

[illegible] 5

[illegible] $= \frac{[illegible]}{0.[illegible]25}$ [illegible]

ω_3^2 [illegible] 0.096[illegible] $=$ [illegible]

ω_3 [illegible] rad./sec.

[illegible]

[illegible]

[illegible] $=$ [illegible] $= \frac{[illegible]}{[illegible]}$ [illegible]

$\omega_4^2 =$ [illegible] $=$ [illegible] $\div 0.0$[illegible]

$\omega_4 =$ [illegible] rad./sec.

[illegible]

DATA FOR CALCULATION OF NORMAL MODES OF VERTICAL VIBRATION

L = 400 ft. E = 1.93 x 10^6 tons/ft.^{2}

Section Stern To Bow	I ft.^{4}	μ $\frac{T sec^2}{Ft^2}$	K	A Ft.^{2}	KAG tons
0-1	617	[illegible]	[illegible]	[illegible]	[illegible]
1-2	1157	0.6271	0.271	7.15	[illegible]
2-3	1536	0.9025	0.220	[illegible]	[illegible]
3-4	1895	1.1300	0.171	12.29	[illegible]
4-5	2146	1.2946	[illegible]	[illegible]	[illegible]
5-6	2314	1.3548	[illegible]	[illegible]	[illegible]
6-7	[illegible]	1.4568	[illegible]	[illegible]	[illegible]
7-8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8-9	[illegible]	1.9540	[illegible]	[illegible]	[illegible]
9-10	2609	2.0060	[illegible]	[illegible]	[illegible]
10-11	2623	[illegible]	0.137	[illegible]	[illegible]
11-12	2628	1.9159	0.134	[illegible]	[illegible]
12-13	[illegible]	1.6765	0.141	15.97	[illegible]
13-14	2614	1.4242	0.152	[illegible]	[illegible]
14-15	[illegible]	1.2439	[illegible]	[illegible]	[illegible]
15-16	2493	[illegible]	[illegible]	12.50	[illegible]
16-17	2281	0.7112	0.205	[illegible]	[illegible]
17-18	1934	[illegible]	[illegible]	[illegible]	[illegible]
18-19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19-20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE II. [illegible] 57

i(t) and $\beta(t)$ in Megohms as Introduced into Computer

L_0 = 2608 ft. $I = I_0\, i(t)$

μ_0 = 2 0060 tons sec.²/ ft.² $\mu = \mu_0\, \beta(t)$

Stern to Bow Section	i(t)	ΔR MEG Δi(t)	ΔR MEG Breakdown	$\beta(t)$	ΔR MEG $\Delta\beta(t)$	ΔR MEG Breakdown
0-1	0.235	0.235	.235	0.036	[illegible]	[illegible]
1-2	[illegible]	0.204	.046 .15[illegible]	0.315	[illegible]	[illegible]
2-3	0.604	0.165	.133 .122	0.450	[illegible]	[illegible]
3-4	0.720	0.116	.116	[illegible]	[illegible]	[illegible]
4-5	[illegible]	0.095	.045 .050	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	0.072	.033 [illegible]	[illegible]	[illegible]	[illegible]
6-7	[illegible]	0.046	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	0.029	[illegible] .01[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	0.021	.021	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	0.009	.00[illegible] [illegible]	1.000	[illegible]	[illegible]
10-11	[illegible]	0.005	.001 .004	0.96[illegible]	[illegible]	
11-12	1.000	0.002	.002	0.955	$\bar{0}$.013	
12-13	1.000	0		[illegible]	[illegible]	
13-14	0.974	$\bar{0}$.006		0.711	[illegible]	
14-15	[illegible]	$\bar{0}$.005		[illegible]	[illegible]	
[illegible]	[illegible]	$\bar{0}$.041		[illegible]	[illegible]	
16-17	[illegible]	$\bar{0}$.080		[illegible]	[illegible]	
17-18	0.765	$\bar{0}$.103		[illegible]	[illegible]	
[illegible]	0.572	$\bar{0}$.192		[illegible]	[illegible]	
[illegible]	[illegible]	$\bar{0}$.291		[illegible]	[illegible]	
ΔX=20′						

$\left(\frac{E}{KAG} + \frac{1}{A}\right)$, [illegible] $\frac{[illegible]}{L^2}$, $D = $ [illegible] $\left[i(t)\,\beta(t) \right]$

Section	E/KAG $1/FT^2$	$\frac{1}{A}$ $1/FT^2$	$\frac{E}{KAG} + \frac{1}{A}$ $1/FT^2$		$i(t)/\beta(t)$
0-1	1.511	0.187	1 [illegible]		0.022
1-2	1.291	0.140	[illegible]		0.137
2-3	1.222	0.107	[illegible]		0.272
3-4	1.190	0.081	1.[illegible]		[illegible]
4-5	1.188	0.071	1.259		[illegible]
5-6	1.222	0.073	1.21[illegible]		[illegible]
6-7	[illegible]	0.073	[illegible]		[illegible]
7-8	1.124	0.072	[illegible]		[illegible]
8-9	1.111	0.069	[illegible]		[illegible]
9-10	1.163	0.067	[illegible]		[illegible]
10-11	1.247	0.068	[illegible]		[illegible]
11-12	1.250	0.067	[illegible]		[illegible]
12-13	1.111	0.062	[illegible]		[illegible]
13-14	1.062	0.065	[illegible]		0.7[illegible]
14-15	1.147	0.077	[illegible]		[illegible]
15-16	1.076	0.080	[illegible]		[illegible]
16-17	1.033	0.085	[illegible]		[illegible]
17-18	1.097	0.078	[illegible]		[illegible]
18-19	1.279	0.125	[illegible]		[illegible]
19-20	1.730	0.182	[illegible]		[illegible]
			$\Sigma =$ [illegible]		$\Sigma =$ [illegible]
			$B_{AVE.} =$ [illegible]		AVE. = [illegible]

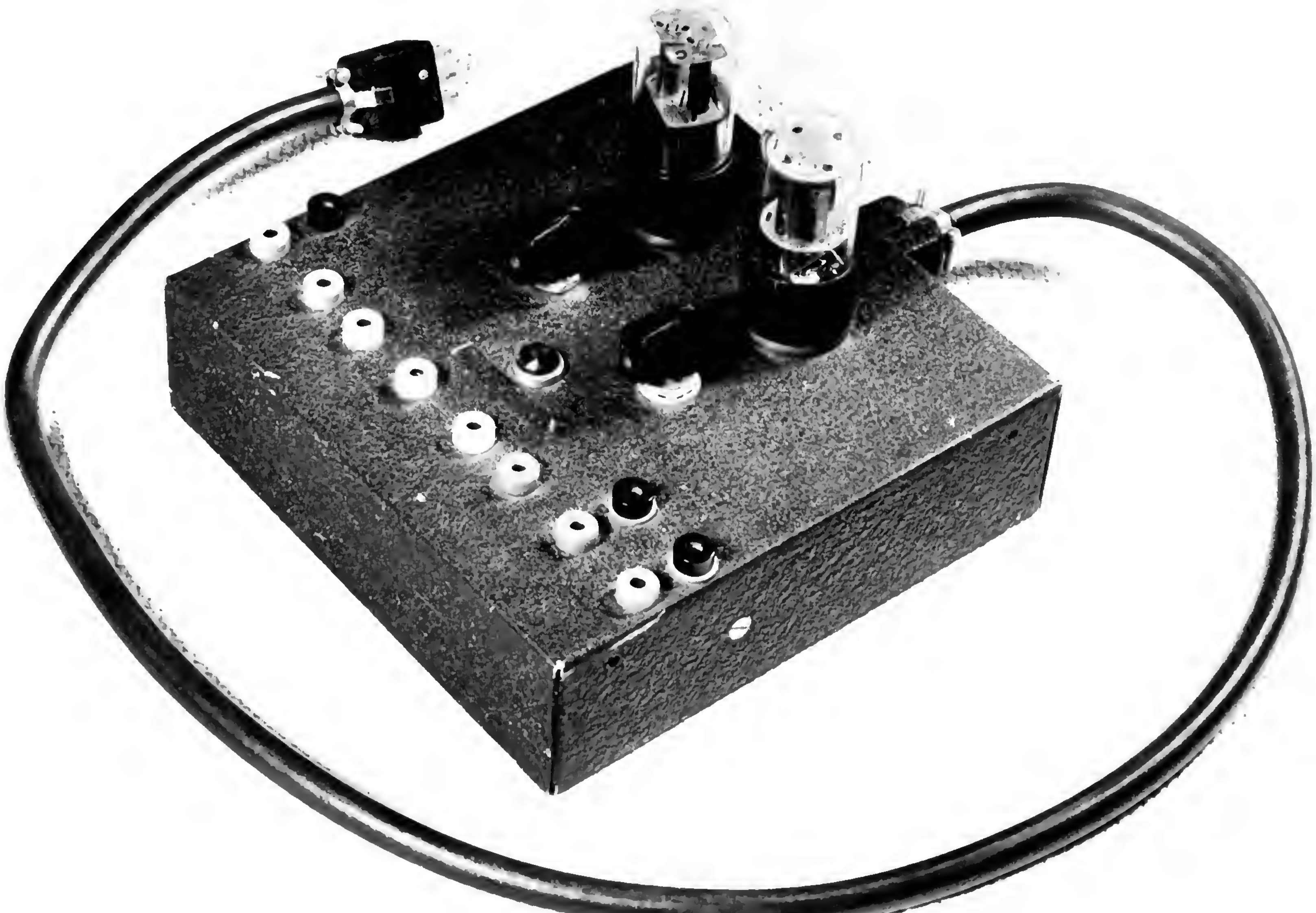
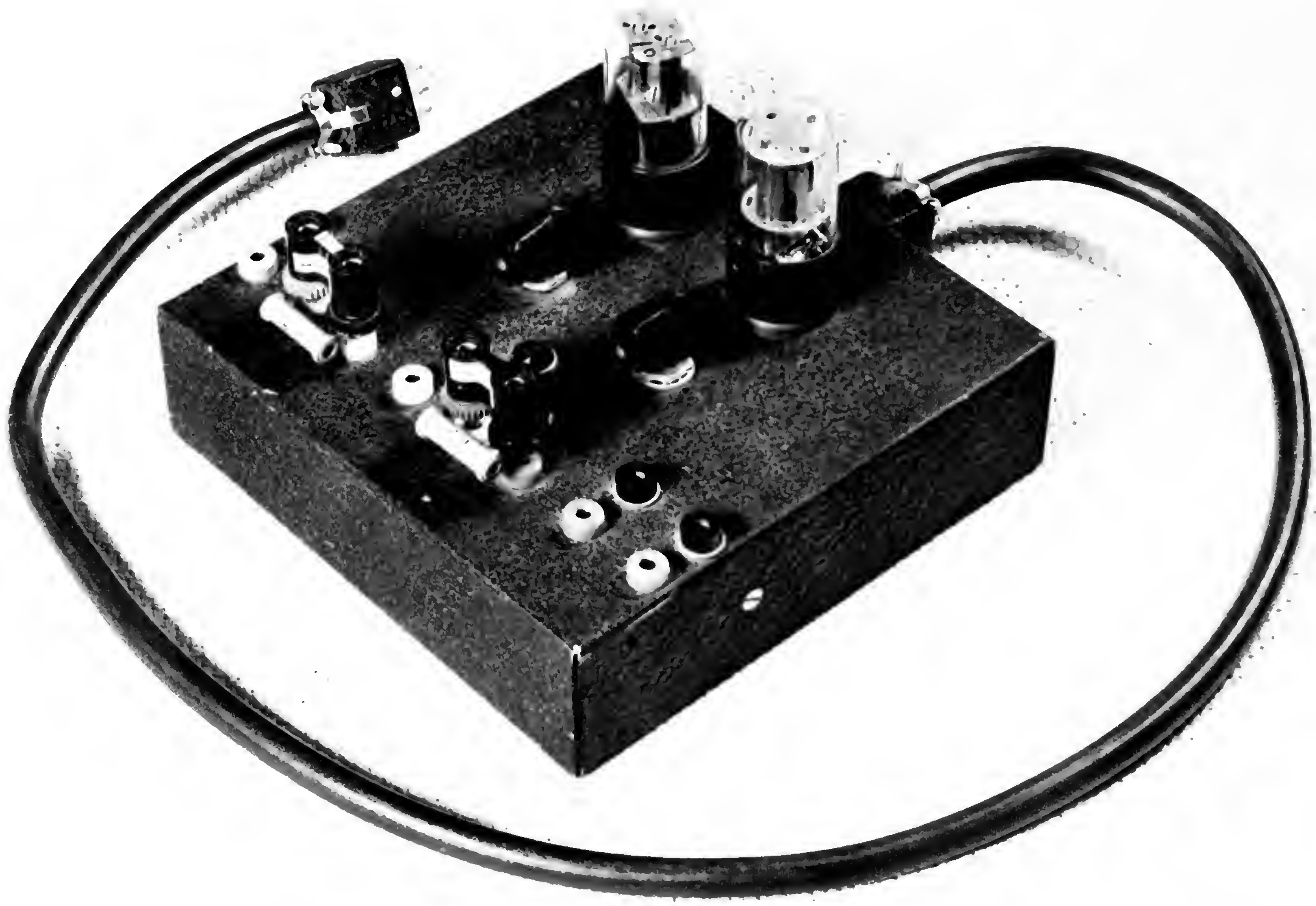


Figure 2 D. C. Amplifier Chassis

Figure 3

D.C. Amplifier Set Up as a Multiplier.



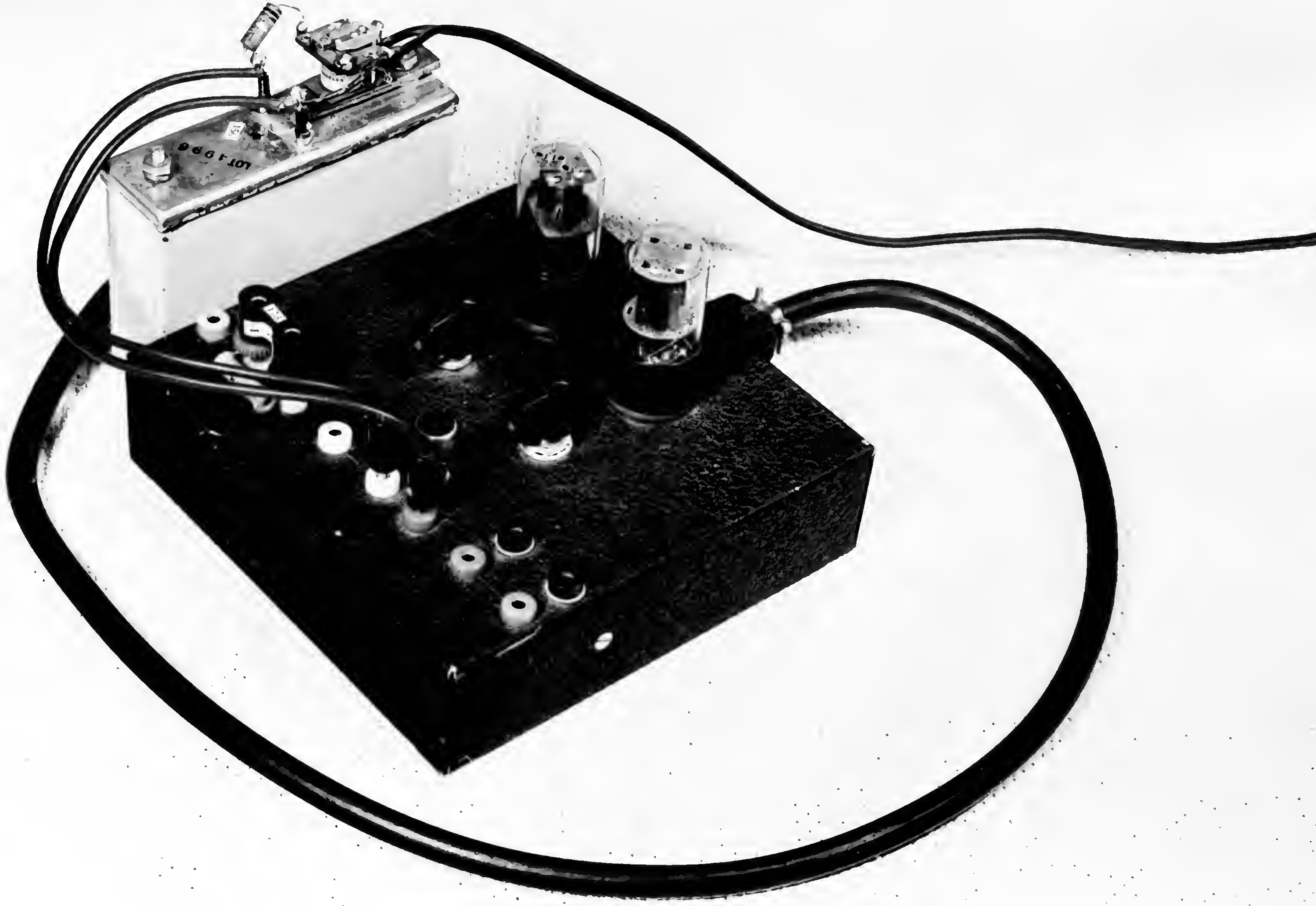


Figure 4 D. C. Amplifier Set Up as an Integrator

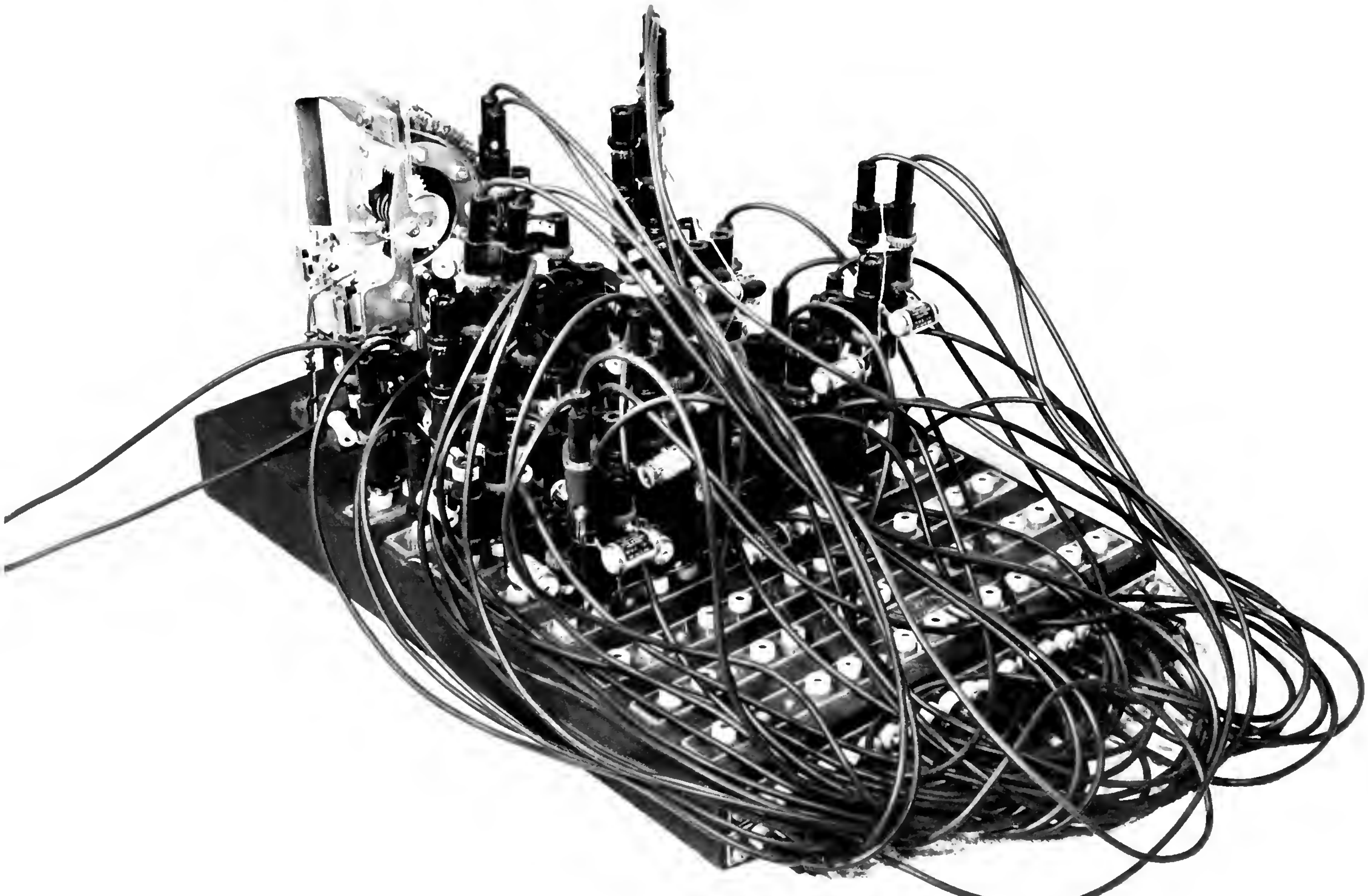


Figure 5

Stepping Relay and Resistor Panel Unit.

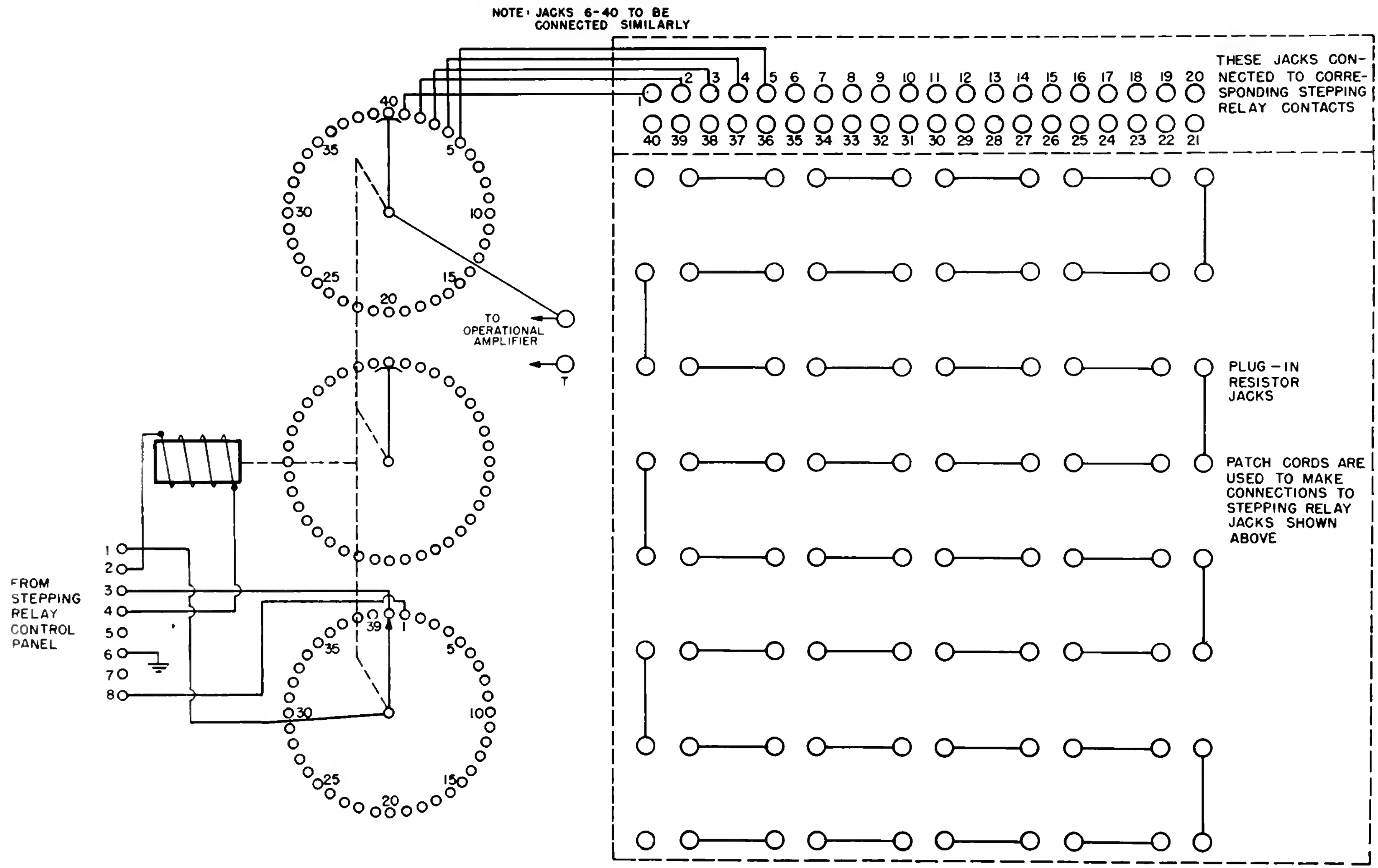
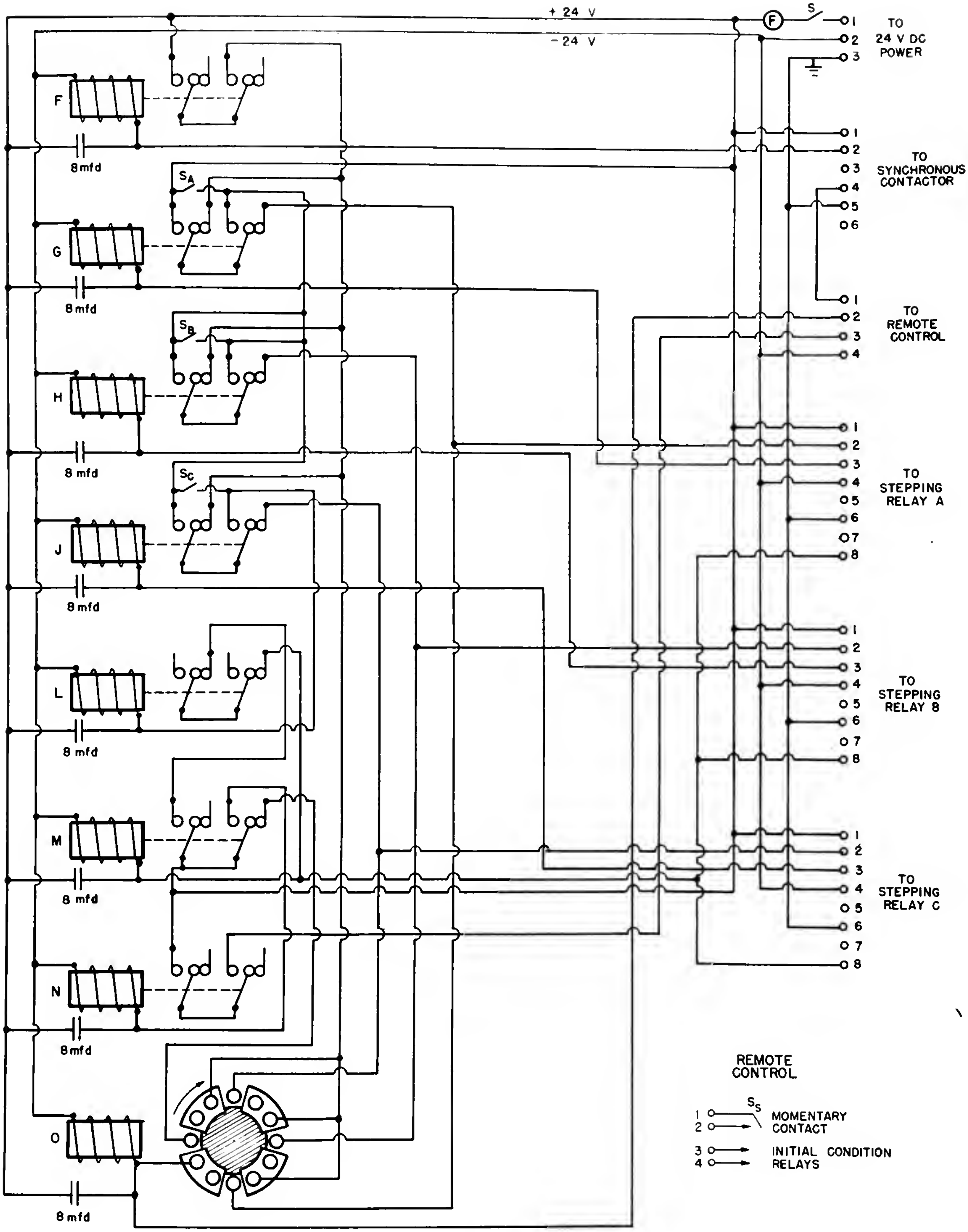


Figure 6.
STEPPING RELAY AND PLUG-IN RESISTOR CIRCUIT

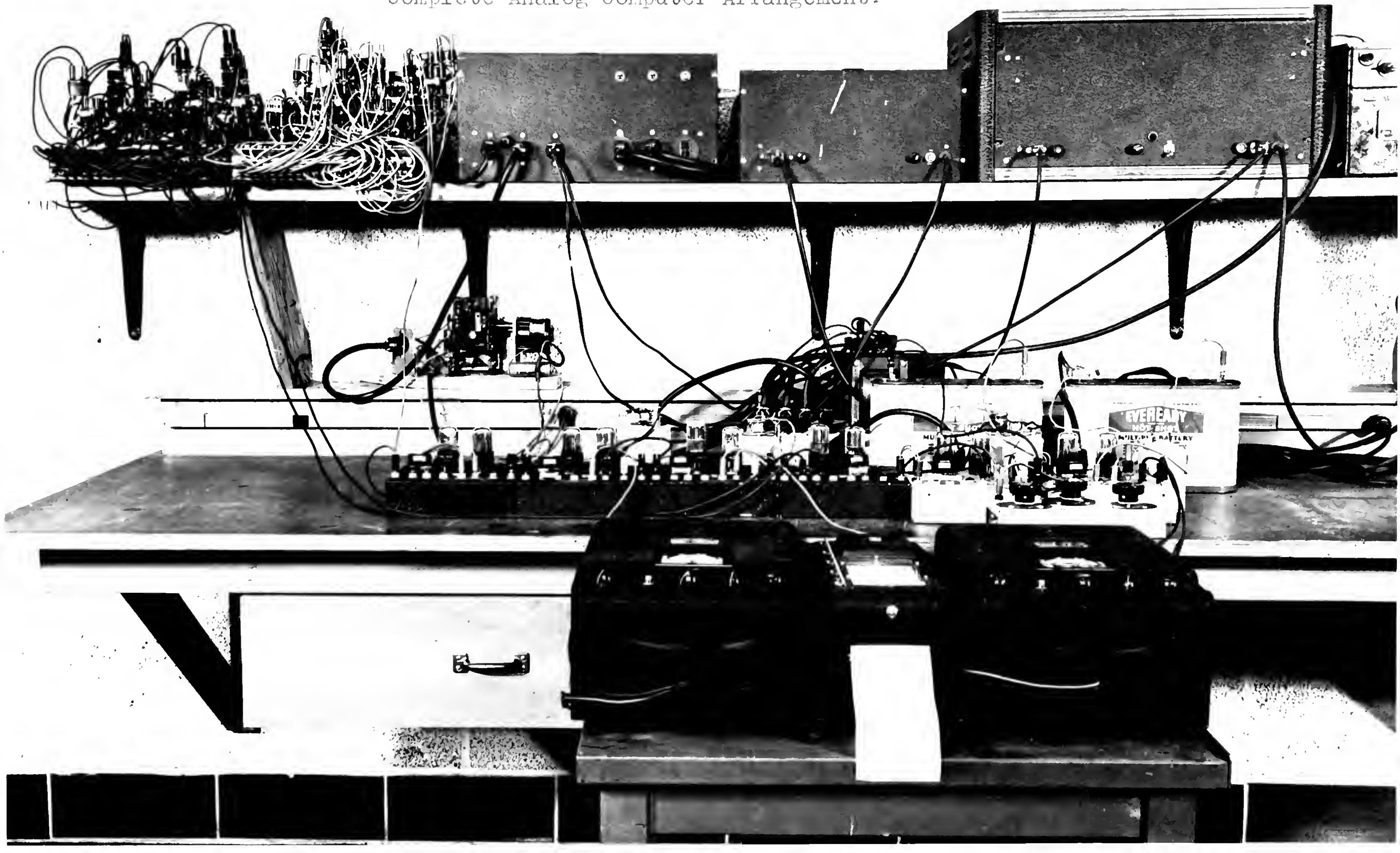


STEPPING RELAY CONTROL CIRCUIT

Figure 7

Figure 8.

Complete Analog Computer Arrangement.



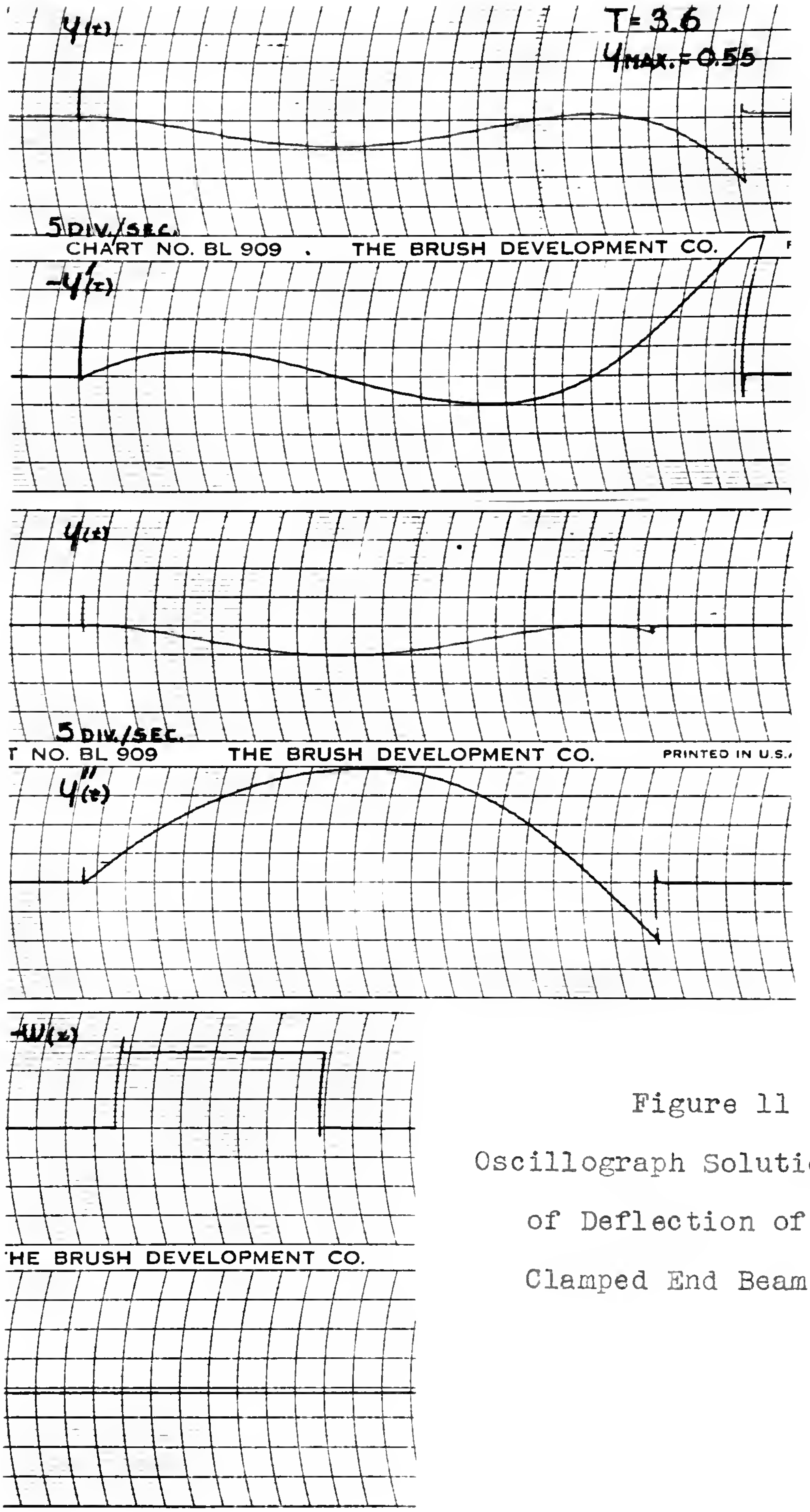


Figure 11

Oscillograph Solutions

of Deflection of

Clamped End Beam.

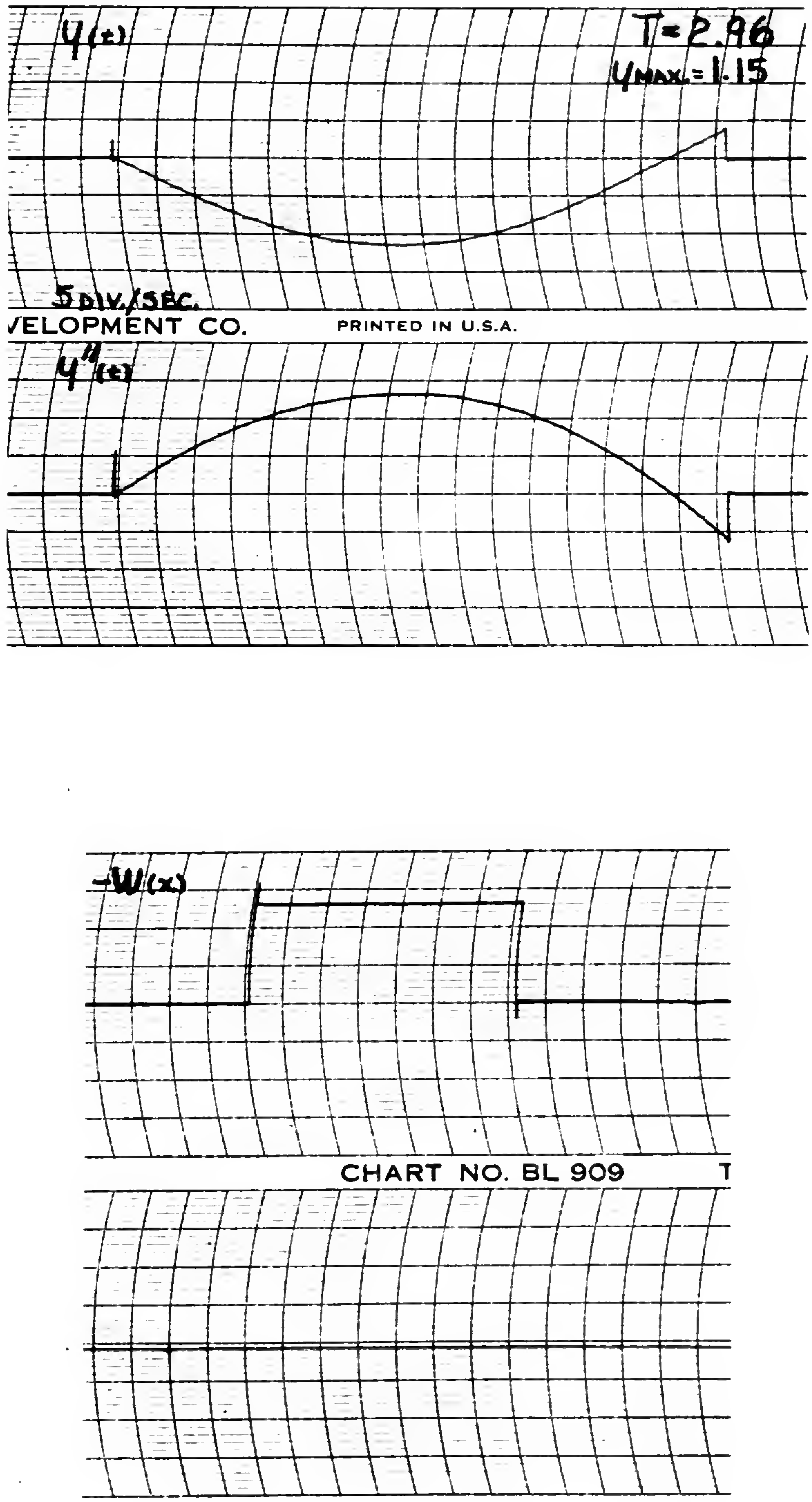


Figure 13

Oscillograph Solution of Deflection of Hinged End Beam.

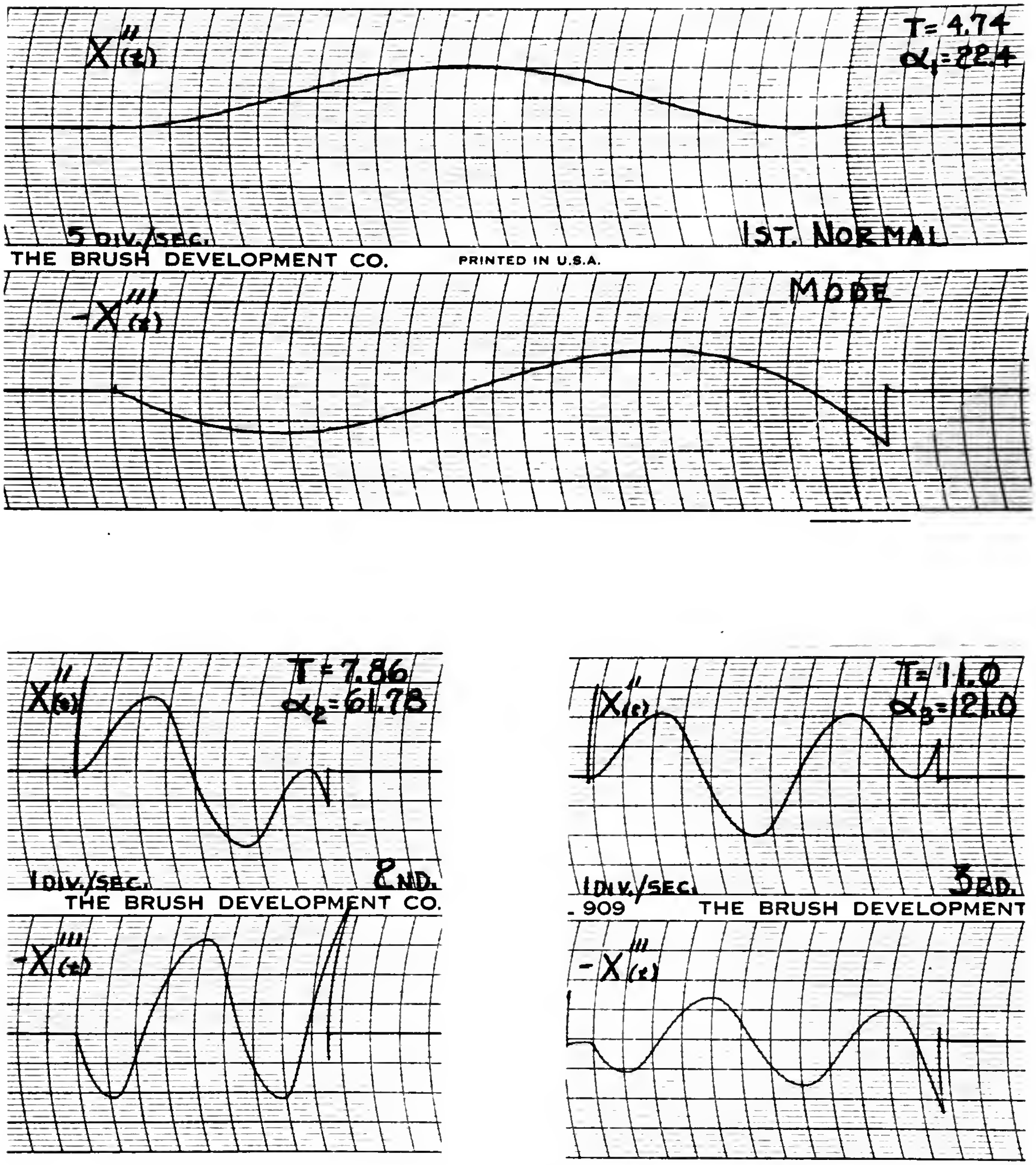
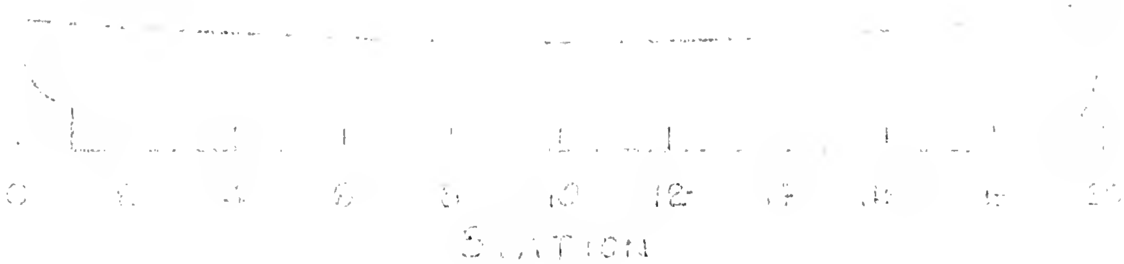
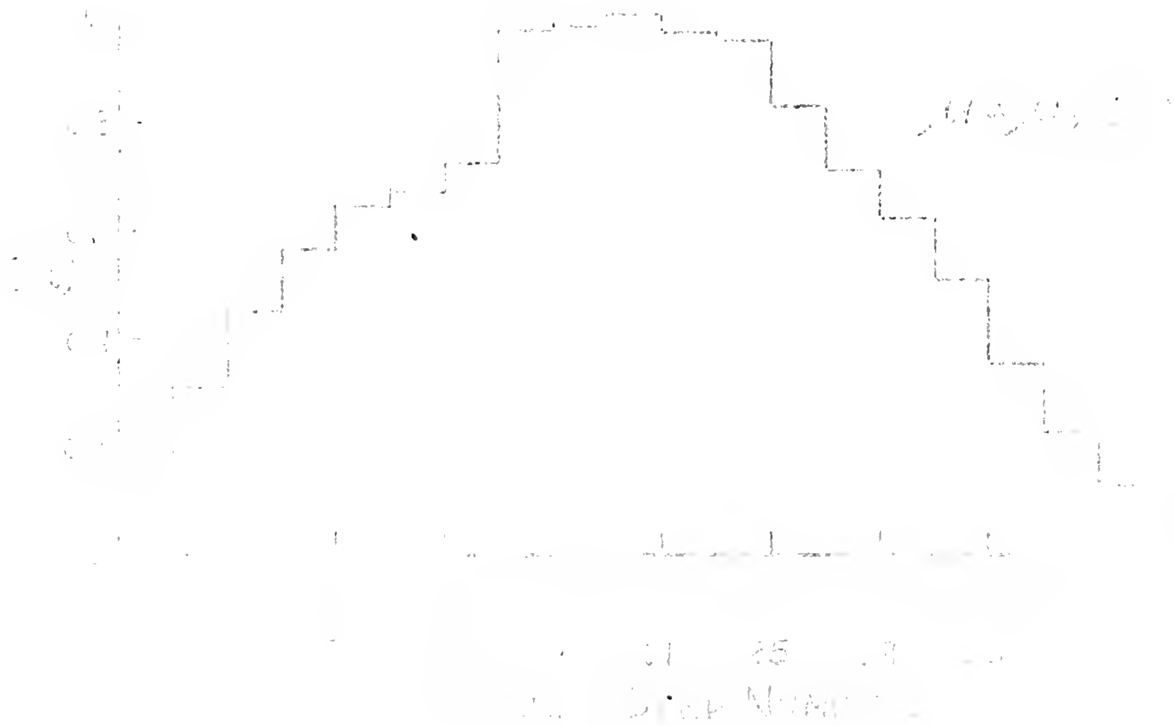


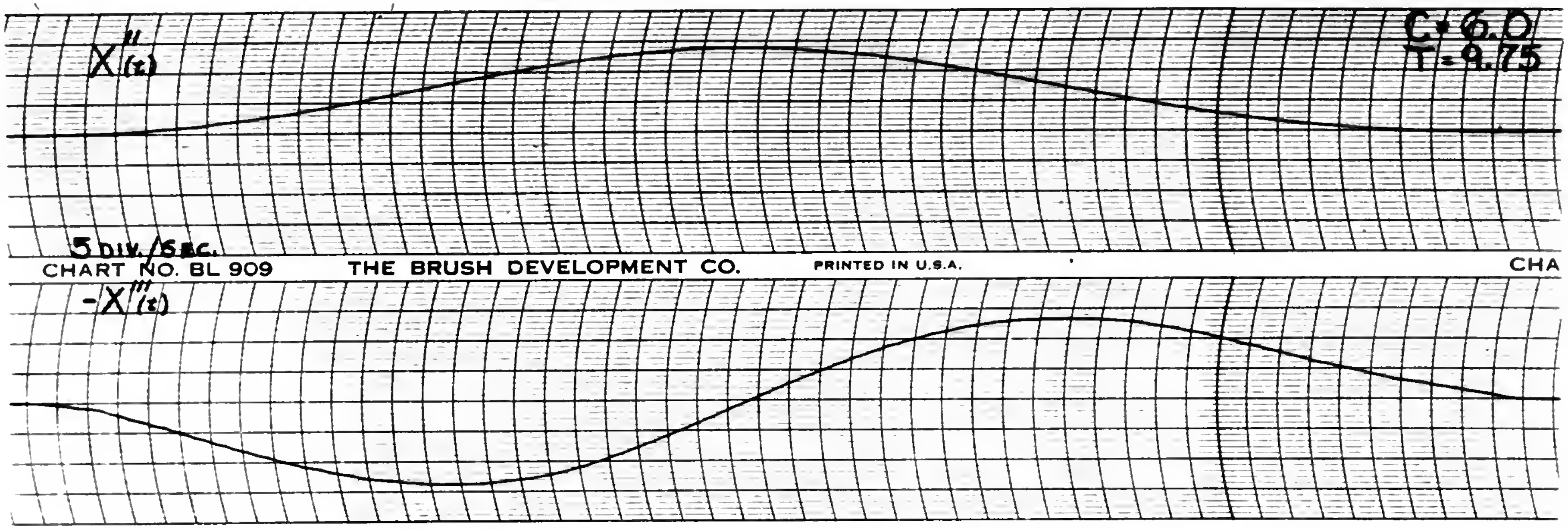
Figure 15

Oscillograph Solutions of First Three Normal Modes,

Uniform Free-free Beam.


STATION





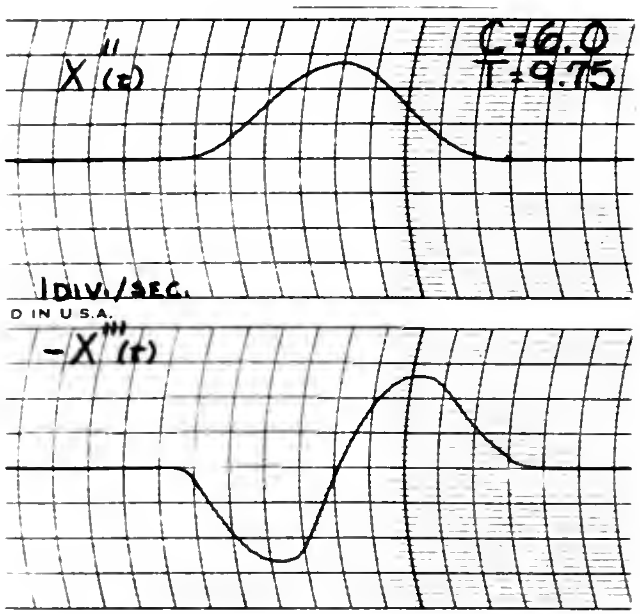
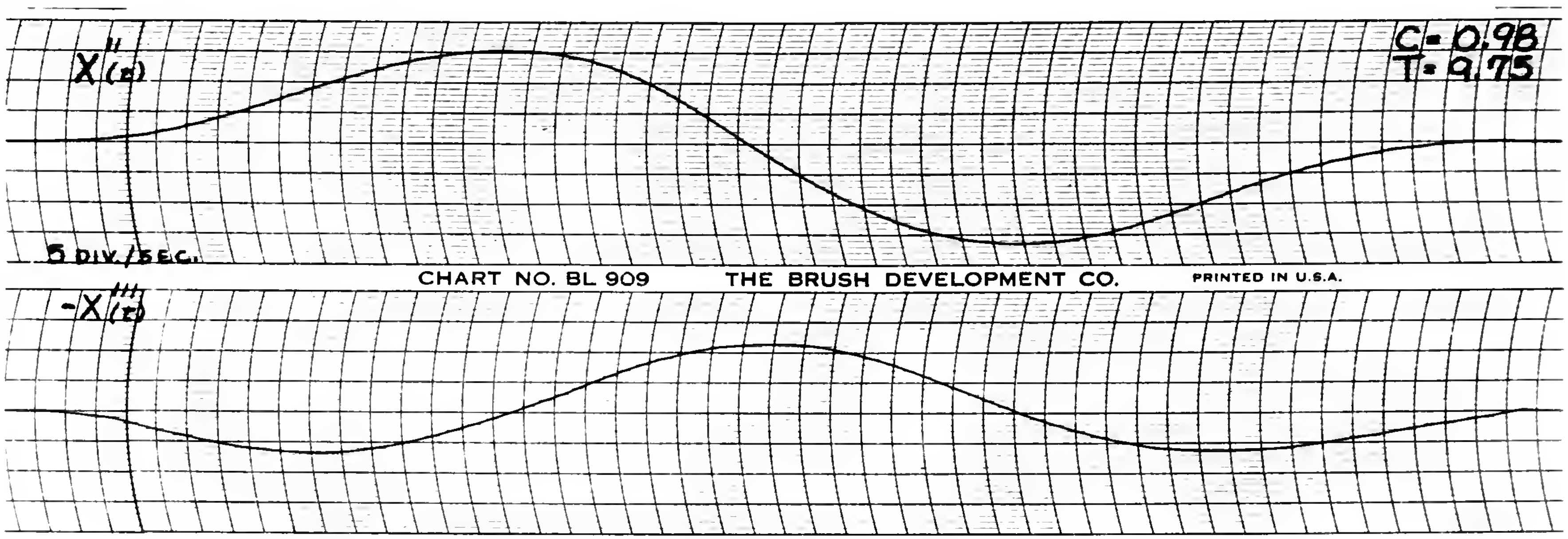


Figure 18

Oscillograph Solution of First Normal Mode

APA 87, Bending Deflections Only.



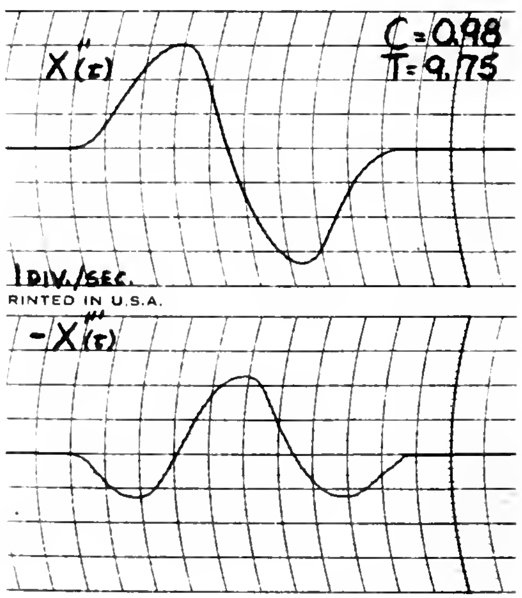


Figure 19

Oscillograph Solutions of Second Normal Mode

APA 87, Bending Deflections Only

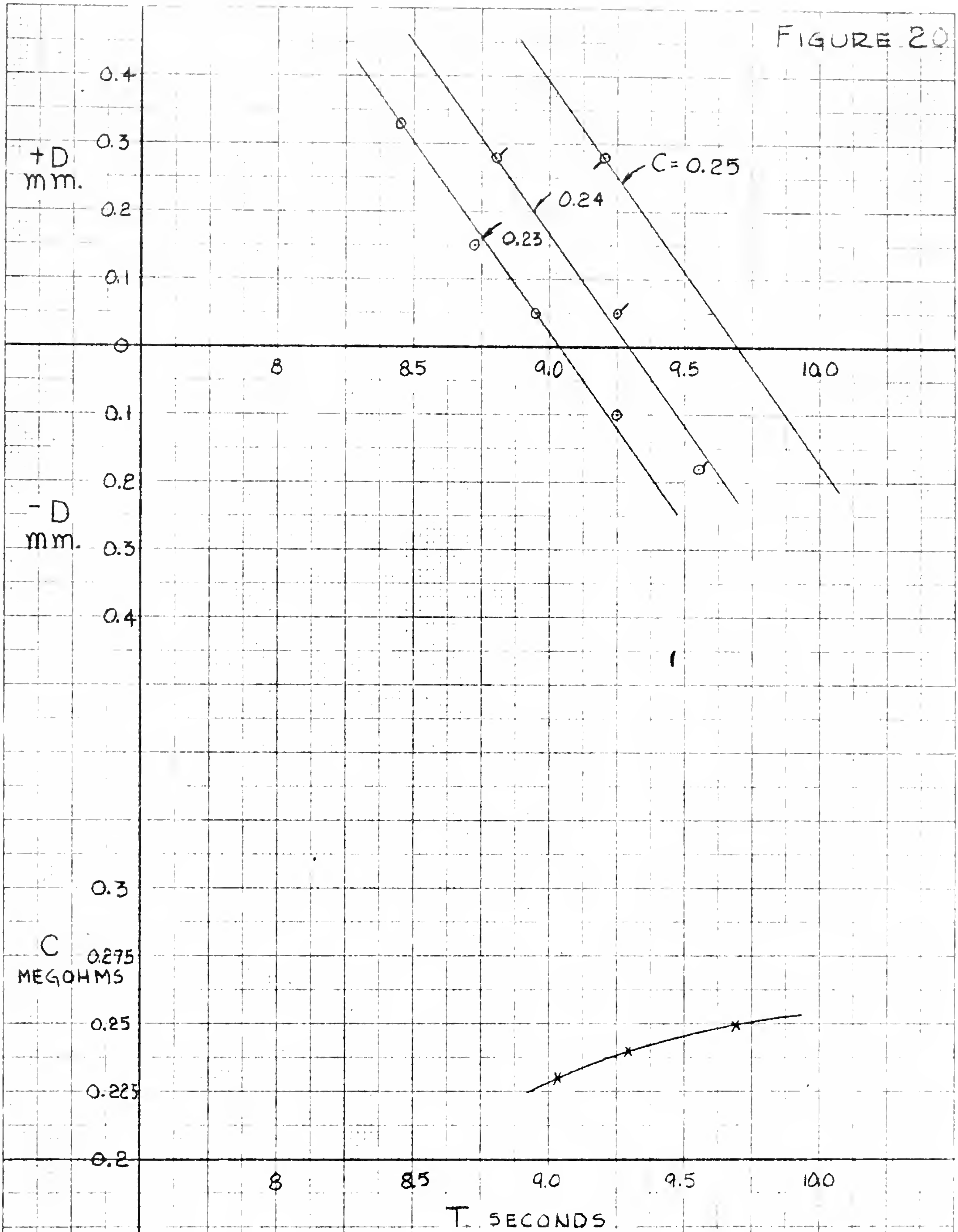
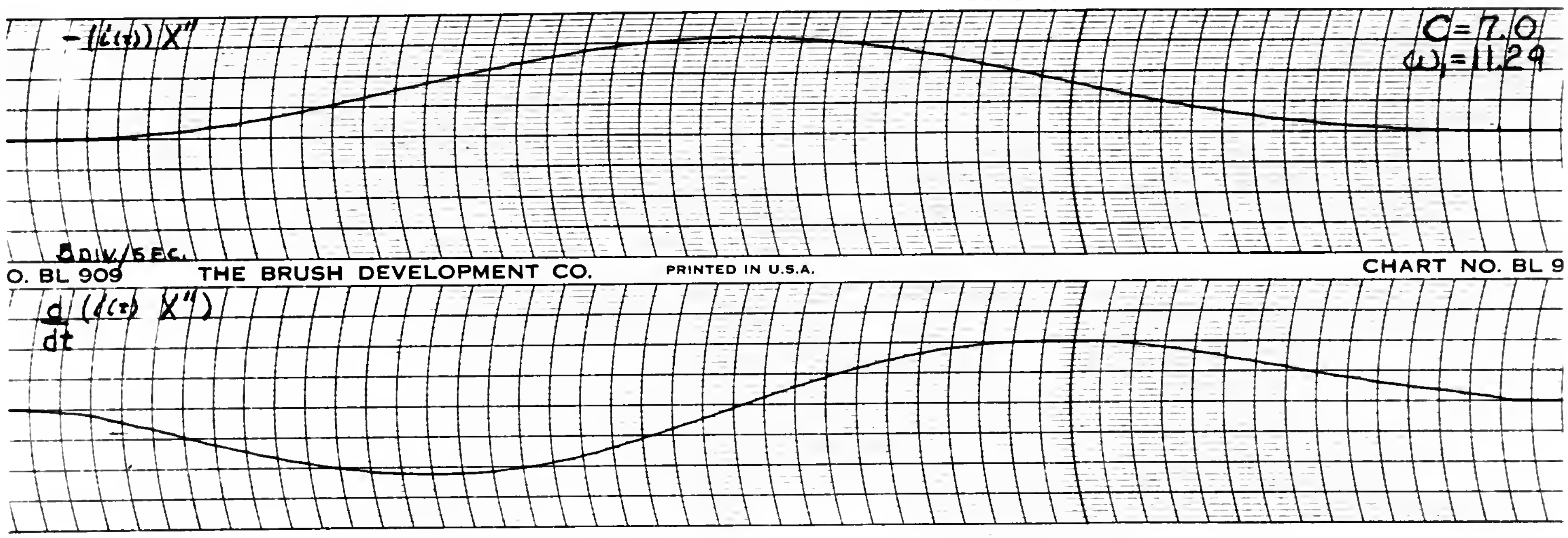


FIGURE 20

GRAPHICAL SOLUTION FROM OSCILLOGRAPH RECORDS OF THIRD MODE, APA 87, BENDING DEFLECTIONS ONLY.

WAE 5/49



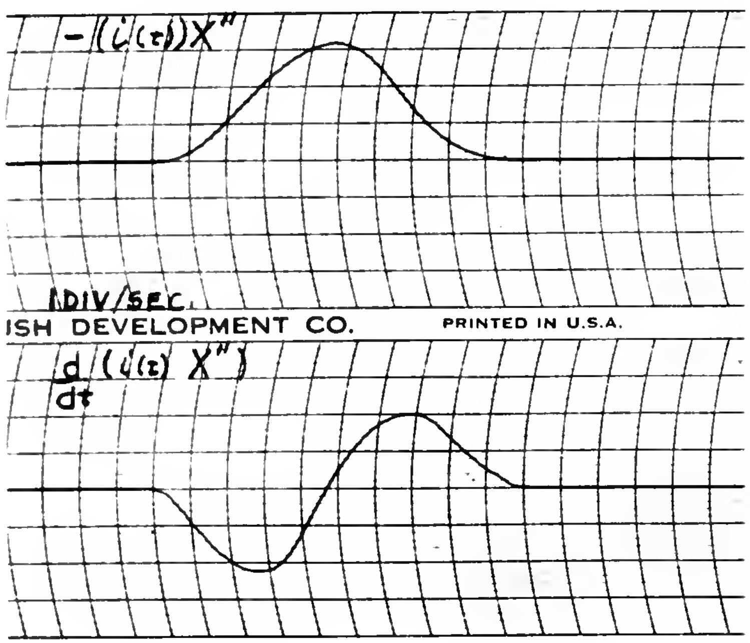
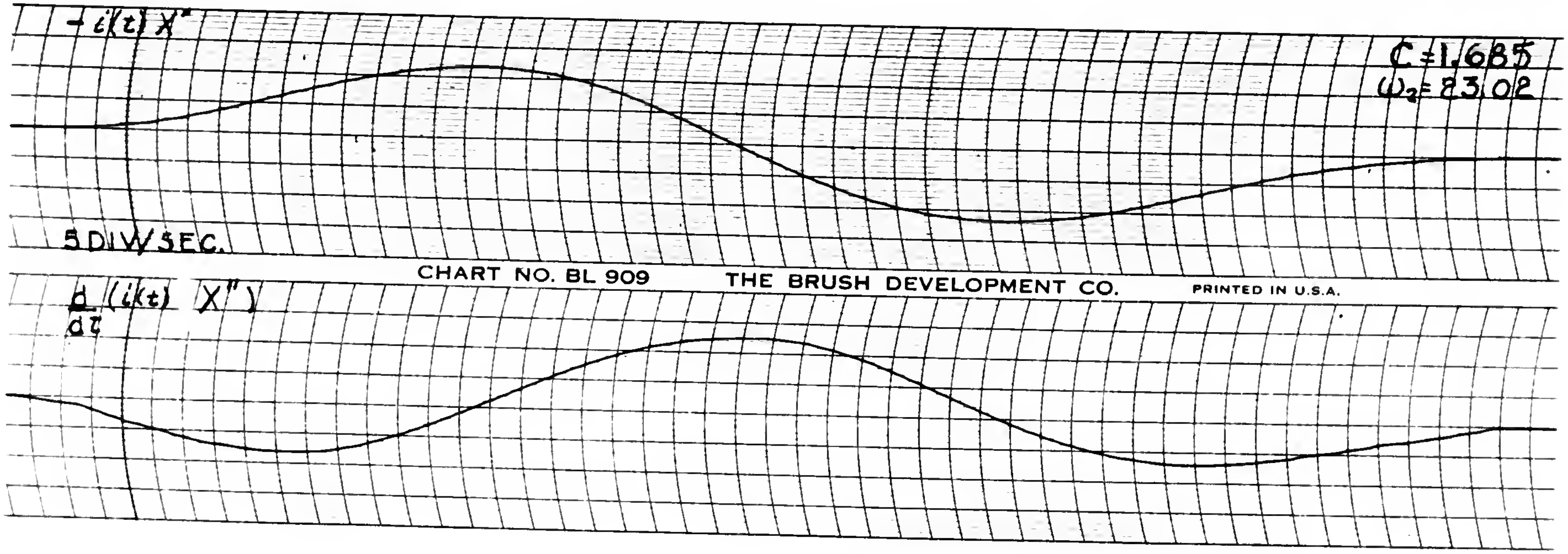


Figure 21

Oscillograph Solution of First Normal Mode, APA 87.
Bending and Shear Deflections and Rotary Inertia.



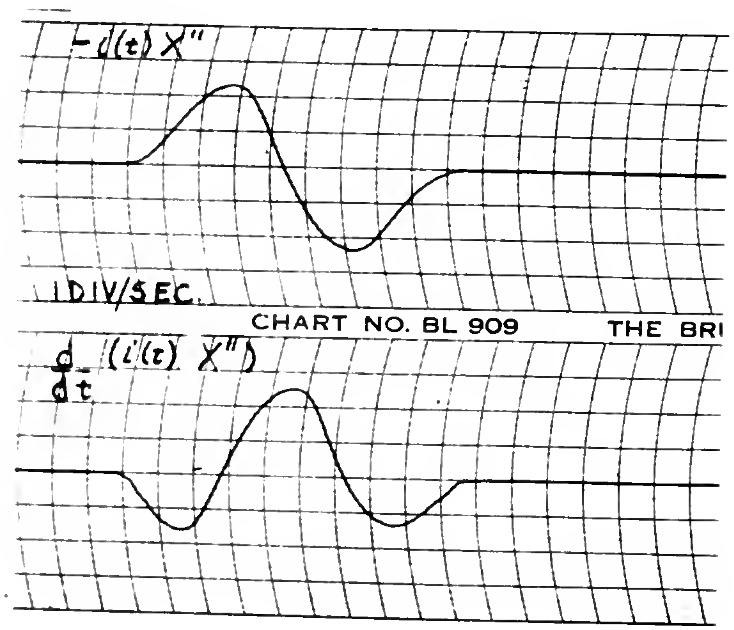
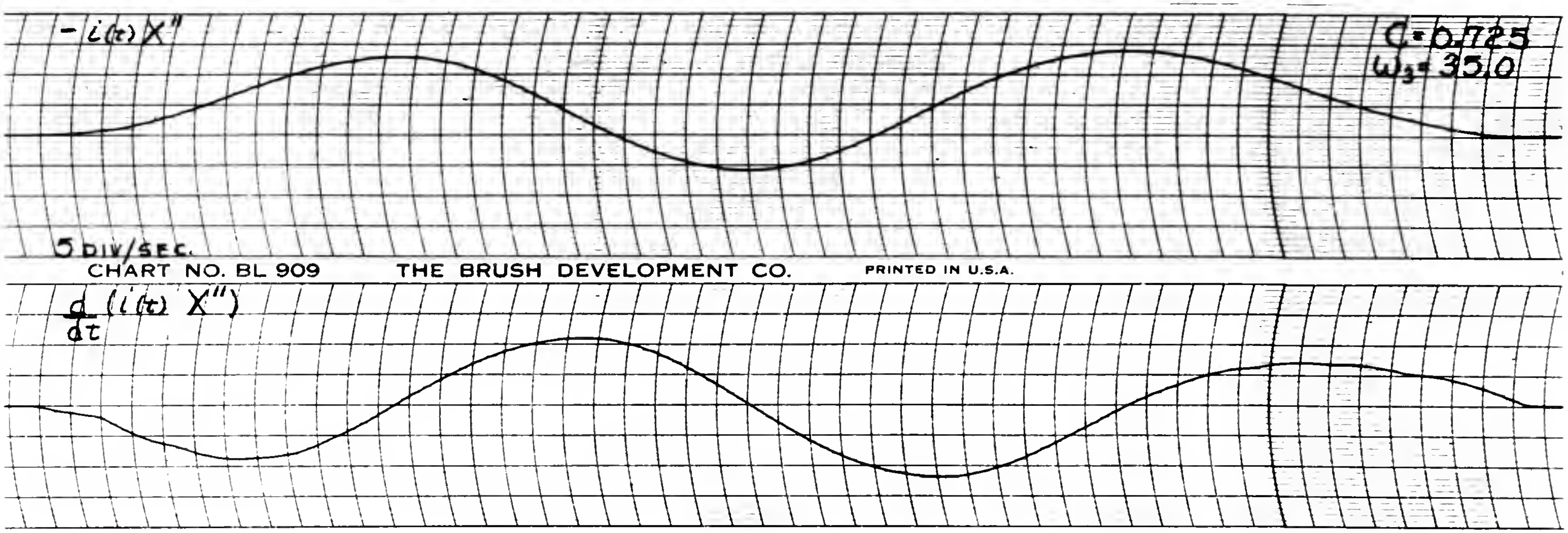


Figure 22

Oscillograph Solution of Second Normal Mode, APA 37, Bending and Shear Deflections and Rotary Inertia.



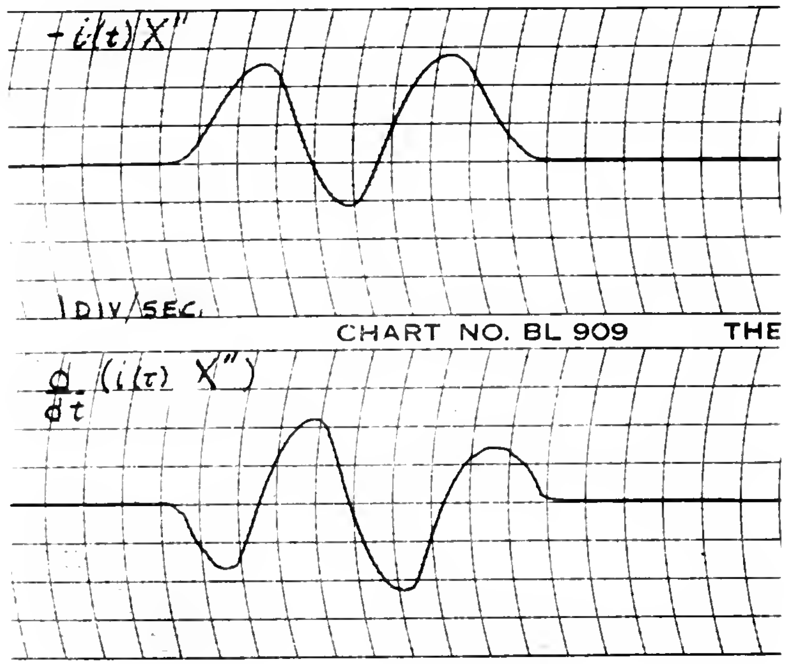


Figure 23

Oscillograph Solution of Third Normal Mode, AFA 37.

Bending and Shear Deflections and Rotary Inertia.

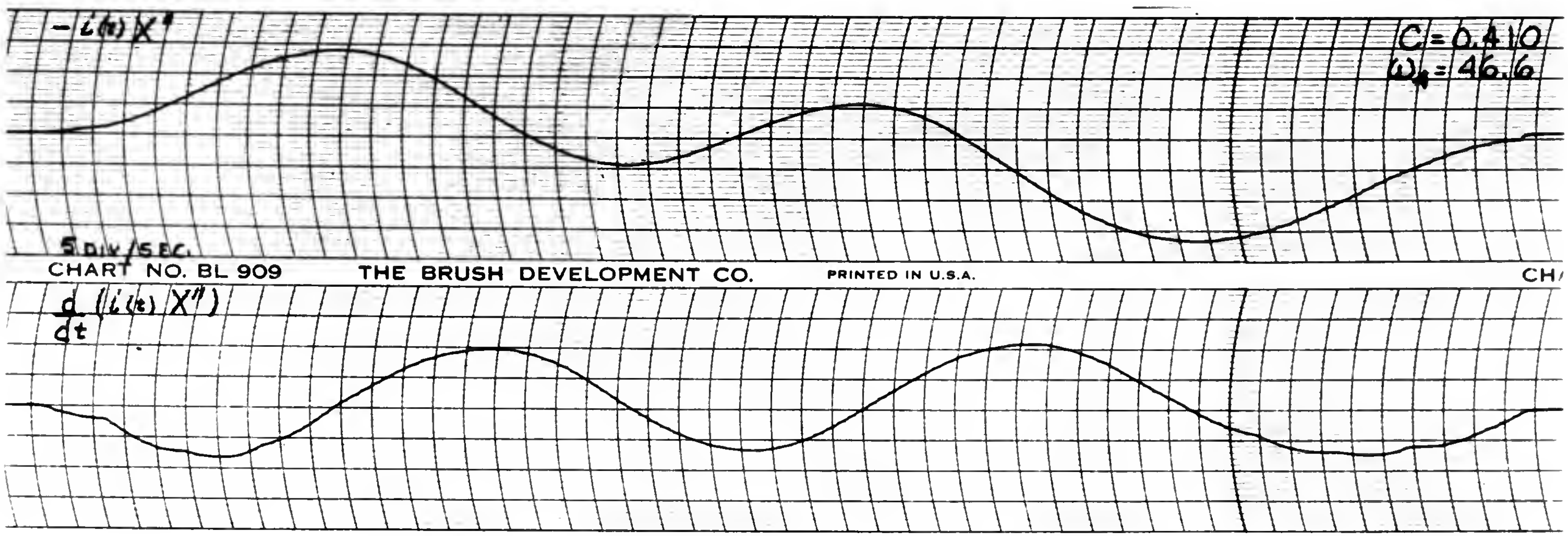


Figure 24

Oscillograph Solution of Fourth Normal Mode, APA 87,

Bending and Shear Deflections and Rotary Inertia.